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CONSTRUCTION OF THE SOLUTION TO THE CAUCHY PROBLEM FOR NONHOMOGENEOUS ORDINARY DIFFERENTIAL EQUATIONS WITH PIECEWISE CONTINUOUS ARGUMENTS

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Abstract. This article addresses the Cauchy problem for first-order linear differential equations with piecewise continuous arguments within the theory of differential equations. The study differs from previous research in this field by considering the nonhomogeneous case, which substantiates the novelty of the results.

Keywords: Piecewise continuous argument, ordinary differential equation, Cauchy problem.

Introduction. Scientific studies on ordinary differential equations with piecewise continuous arguments have been conducted by numerous researchers, including O. Arino, I. Ciyöri, O. Ladas, Y. G. Sfikas, Y. Kitamura, T. Kusano, L. Debnath, K. Cooke, M. U. Akhmet, Huseyin Bereketoglu, Mehtap Lafchi, J. Winer, and many others. The initial results in this field were published by J. Winer starting from 1991.

Although many problems in this area have been formulated and solved to date, equations involving nonhomogeneous piecewise continuous arguments and the corresponding problems have not been sufficiently investigated in the works of many researchers. This indicates the necessity and continued relevance of research in this direction.

In this study, we examine the solution of the Cauchy problem (1)-(2) for a first-order ordinary differential equation with a nonhomogeneous piecewise continuous argument.

$$v'(t)+av(t)+bv([t])=f([t]) \qquad (1)$$

$$v(0)=c_0. \qquad (2)$$

Here, $a,b\in R$, $t\in[0,\infty)$ and $f([t])$, is a given function.

$$n\le t<n+1,\ v_j'(t)=-av_j(t)-bv_j(n)+f_j(n) \qquad (3)$$

$$v_j'(t)=-av_j(t) \qquad (4)$$

$$v_j(t)=A_je^{-at} \qquad (5)$$

$$A_j=A_j(t),\ A_j(t)=\int_0^t(-bv_j(n)+f_j(n))e^{a\tau}d\tau+A_0 \qquad (6)$$

$$v_j(t)=\frac{-bv_j(n)+f_j(n)}{a}(1-e^{-at})+A_0e^{-at} \qquad (7)$$

$$t=n, v_{nj}(n)=\frac{-bv_{nj}(n)+f_{nj}(n)}{a}(1-e^{-an})+A_0e^{-an}$$

$$A_0=v_{nj}(n)e^{an}+\frac{bv_{nj}(n)-f_{nj}(n)}{a}(e^{an}-1) \qquad (8)$$

$$v_{nj}(t)=\left(e^{-a(t-n)}-\frac{b}{a}(1-e^{-a(t-n)})\right)v_{nj}(n)+\frac{f_{nj}(n)}{a}(1-e^{-a(t-n)}) \qquad (9)$$

$$W(t)=e^{-at}-\frac{b}{a}(1-e^{-at}),\ F(t,n)=\frac{f_{nj}(n)}{a}(1-e^{-a(t-n)}).$$

$$v_{nj}(t)=W(t-n)v_{nj}(n)+F(t,n) \qquad (10)$$

$$v_{nj}(n+1)=v_{n+1j}(n+1) \quad (11)$$

$$t=n+1,\ v_{n+1,j}(n+1)=W(1)v_{nj}(n)+F(n+1,n) \quad (12)$$

$$v_{nj}(n)=W^n(1)v_{0j}(0)+\sum_{k=1}^{n}W^{n-k}(1)F(k,k-1) \quad (13)$$

$$v_{nj}(t)=W(t-n)\left[W^n(1)v_{0j}(0)+\sum_{k=1}^{n}W^{n-k}(1)F(k,k-1)\right]+F(t,n) \quad (14)$$

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