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LOCAL LINEAR TRIGONOMETRIC FUNDAMENTAL SPLINES FOR APPROXIMATING SOME GEOMETRIC CURVES IN THE PLANE

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Abstract. In this work, new local trigonometric fundamental splines are constructed for the integration of some geometric curves on a plane. In this case, we use the coefficients of the trigonometric optimal interpolation formula, constructed using the Sobolev method in a known Hilbert space of differentiable functions. In addition, we will present and prove the theorem expressing their main property.

Keywords. Approximation, interpolation, basis functions, fundamental splines, optimal interpolation formula, geometric curves, Sobolev method.

It is well known that, at present, function approximation is widely used in obtaining approximate solutions to many practical problems. In functions approximation, the construction of interpolatory fundamental splines is of great importance. For this purpose, in the present work we consider the problem of constructing a fundamental spline corresponding to an optimal interpolation formula defined in a certain Hilbert space.

In this work [1], a trigonometric optimal interpolation formula was constructed in the Hilbert space

$$K_{2,\omega}^{(2)} = \{\varphi : [0,1] \to R \mid \varphi' - \text{abs. con. and } \varphi'' \quad L_2(0,1)\},$$

and using its coefficients, the following set of basis functions $\mu_i(x)$ $(i = 0,1,\ldots,n)$ was created:

$$\mu_0(x) = \begin{array}{l} \dfrac{\sin(\omega x - \omega x_1)}{\sin(\omega x_0 - \omega x_1)}, x_0 \quad x < x_1, \\ 0, x_1 \quad x \quad 1, \end{array} \tag{1}$$

$$\mu_i(x) = \begin{array}{l} 0, x_0 \quad x < x_{i-1}, \\ \dfrac{\sin(\omega x - \omega x_{i-1})}{\sin(\omega x_i - \omega x_{i-1})}, x_{i-1} \quad x < x_i, \\ \dfrac{\sin(\omega x - \omega x_{i+1})}{\sin(\omega x_i - \omega x_{i+1})}, x_i \quad x < x_{i+1}, \\ 0, x_{i+1} \quad x \quad 1, \end{array} \quad (i=1,2,\ldots,n-1) \tag{2}$$

and

$$\mu_n(x) = \begin{array}{l} 0, x_0 \quad x < x_{n-1}, \\ \dfrac{\sin(\omega x - \omega x_{n-1})}{\sin(\omega x_n - \omega x_{n-1})}, x_{n-1} \quad x \quad x_n, \end{array} \tag{3}$$

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where $0=x_0<x_1<...<x_n=1,\ x_i=ih,\ h=\frac{1}{n},\ i=0,1,...,n$ and ω $\backslash\{0\}$.

Now, to geometrically represent the coefficients $\mu_i(x)\ (i=0,1,...,n)$, we present their graphs. The graphs of the coefficients $\mu_i(x)\ (i=0,1,2,3,4,5)$ for the case $n=5$ are shown in Fig. 1.

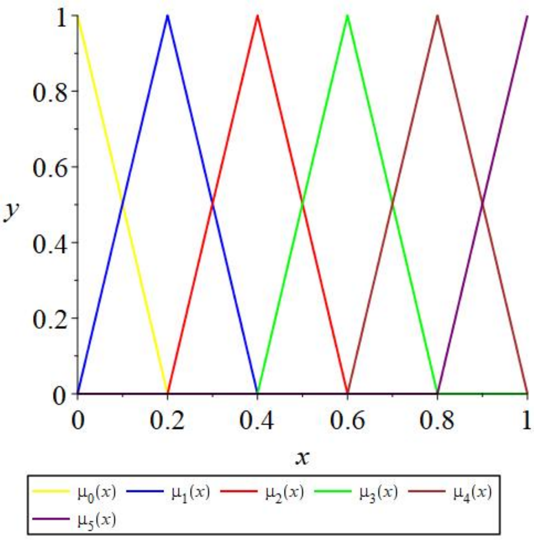

Figure 1. This figure shows the graphs of $\mu_0(x),\mu_1(x),\mu_2(x),\mu_3(x),\mu_4(x)$ and $\mu_5(x)$ when $n=5$ and $\omega=1$ in expressions $(1)-(3)$, respectively.

The following theorem holds for these two-point local trigonometric basis functions $\mu_i(x)\ (i=0,1,...,n)$ [1].

Theorem 1. (Theorem on the main property) For local basis functions $\mu_i(x)\ (i=0,1,...,n)$, the following relations are valid:

$$\sum_{i=0}^{n}\mu_i(x)\sin(\omega x_i)=\sin(\omega x),$$

$$\sum_{i=0}^{n}\mu_i(x)\cos(\omega x_i)=\cos(\omega x).$$

The proof of Theorem 1 follows from the results obtained in work [1].

From Theorem 1 we conclude that the linear combinations of the basis functions $\mu_i(x)\ (i=0,1,...,n)$ exact for functions $\sin(\omega x)$ and $\cos(\omega x)$.

Remark 1. For the basis functions $\mu_i(x)\ (i=0,1,...,n)$, defined by the expressions $(1)-(3)$, the relation

$$\mu_i(x_j)=\begin{cases}1, i=j,\\ 0, \text{otherwise}\end{cases}$$

i.e.,

$$\mu_i\left(x_j\right)=\delta_{ij}, i=0,1,...,n, j=0,1,...,n,$$

holds, where δ_{ij} is the Kroniker symbol [3].

The validity of Remark 1 is evident directly from Fig. 1.

Remark 2. The optimal coefficients $\mu_i\left(x\right)$ $\left(i=0,1,...,n\right)$ are local linear trigonometric fundamental splines in the Hilbert space $K_{2,\omega}^{(2)}$.

The validity of Remark 2 follows from the definition of the fundamental spline [2] and Remark 1.

It should be noted that using fundamental splines $\mu_i\left(x\right)$ $\left(i=0,1,...,n\right)$, it is possible to approximate some geometric curves on a plane with high accuracy.

Conclusion

In this work, local trigonometric basis functions were constructed using the coefficients of the optimal interpolation formula constructed in the Hilbert space $K_{2,\omega}^{(2)}$. It has been proven that these basis functions are fundamental splines. Also, the theorem expressing the main property of these fundamental splines was proven.

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